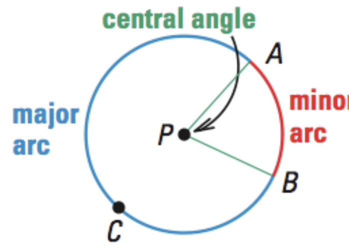
Geometry 10.2 Arcs & Chords Notes (pp 603-606)

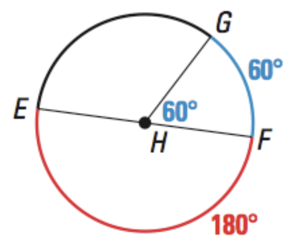
In a plane, an angle whose vertex is the center of a circle is a **central angle** of the circle.

If the measure of a central angle, $\angle APB$, is less than $180°$, then A and B and the points of $\odot P$ in the interior of $\angle APB$ form a **minor arc** of the circle. The points A and B and the points of $\odot P$ in the *exterior* of $\angle APB$ form a **major arc** of the circle. If the endpoints of an arc are the endpoints of a diameter, then the arc is a **semicircle**.



NAMING ARCS Arcs are named by their endpoints. For example, the minor arc associated with $\angle APB$ above is $\overset{\frown}{AB}$. Major arcs and semicircles are named by their endpoints and by a point on the arc. For example, the major arc associated with $\angle APB$ above is $\overset{\frown}{ACB}$. $\overset{\frown}{EGF}$ below is a semicircle.

MEASURING ARCS The **measure of a minor arc** is defined to be the measure of its central angle. For instance, $m\overset{\frown}{GF} = m\angle GHF = 60°$. "$m\overset{\frown}{GF}$" is read "the measure of arc GF." You can write the measure of an arc next to the arc. The measure of a semicircle is $180°$.



The **measure of a major arc** is defined as the difference between $360°$ and the measure of its associated minor arc. For example, $m\overset{\frown}{GEF} = 360° - 60° = 300°$. The measure of a whole circle is $360°$.

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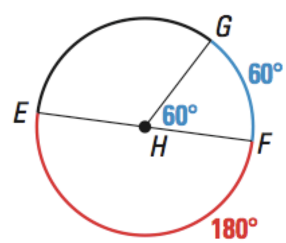

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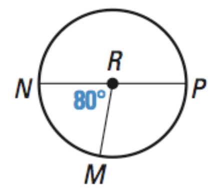
EXAMPLE 1 Finding Measures of Arcs

Find the measure of each arc of $\odot R$.

a. $\overset{\frown}{MN}$

b. $\overset{\frown}{MPN}$

c. $\overset{\frown}{PMN}$



SOLUTION

a. $\overset{\frown}{MN}$ is a minor arc, so $m\overset{\frown}{MN} = m\angle MRN = 80°$

b. $\overset{\frown}{MPN}$ is a major arc, so $m\overset{\frown}{MPN} = 360° - 80° = 280°$

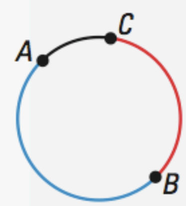
c. $\overset{\frown}{PMN}$ is a semicircle, so $m\overset{\frown}{PMN} = 180°$

POSTULATE

POSTULATE 26 *Arc Addition Postulate*

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

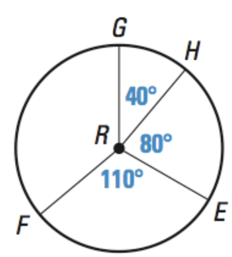
$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$



EXAMPLE 2 Finding Measures of Arcs

Find the measure of each arc.

a. $\overset{\frown}{GE}$ **b.** $\overset{\frown}{GEF}$ **c.** $\overset{\frown}{GF}$



SOLUTION

a. $m\overset{\frown}{GE} = m\overset{\frown}{GH} + m\overset{\frown}{HE} = 40° + 80° = 120°$

b. $m\overset{\frown}{GEF} = m\overset{\frown}{GE} + m\overset{\frown}{EF} = 120° + 110° = 230°$

c. $m\overset{\frown}{GF} = 360° - m\overset{\frown}{GEF} = 360° - 230° = 130°$

....

Two arcs of the same circle or of congruent circles are **congruent arcs** if they have the same measure. So, two minor arcs of the same circle or of congruent circles are congruent if their central angles are congruent.

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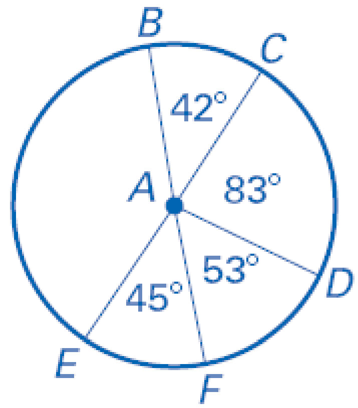
<u>Guided Practice:</u> Find the measure of each arc of $\odot A$.

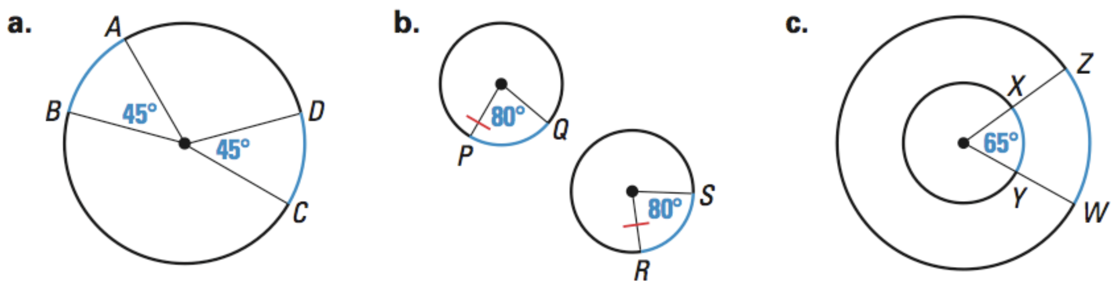
10. $\overparen{BD}$ **11.** $\overparen{BE}$

12. $\overparen{BED}$



EXAMPLE 3 ***Identifying Congruent Arcs***

Find the measures of the blue arcs. Are the arcs congruent?



SOLUTION

a. $\overparen{AB}$ and $\overparen{DC}$ are in the same circle and $m\overparen{AB} = m\overparen{DC} = 45°$. So, $\overparen{AB} \cong \overparen{DC}$.

b. $\overparen{PQ}$ and $\overparen{RS}$ are in congruent circles and $m\overparen{PQ} = m\overparen{RS} = 80°$. So, $\overparen{PQ} \cong \overparen{RS}$.

c. $m\overparen{XY} = m\overparen{ZW} = 65°$, but $\overparen{XY}$ and $\overparen{ZW}$ are not arcs of the same circle or of congruent circles, so $\overparen{XY}$ and $\overparen{ZW}$ are *not* congruent.

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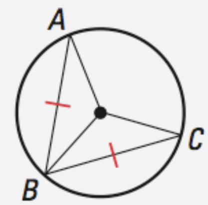
THEOREMS ABOUT CHORDS OF CIRCLES

THEOREM 10.4

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

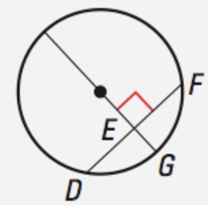
$\overset{\frown}{AB} \cong \overset{\frown}{BC}$ if and only if $\overline{AB} \cong \overline{BC}$.



THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

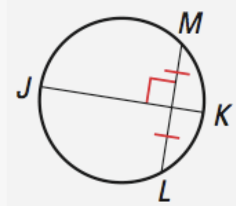
$\overline{DE} \cong \overline{EF}$, $\overset{\frown}{DG} \cong \overset{\frown}{GF}$



THEOREM 10.6

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

$\overline{JK}$ is a diameter of the circle.



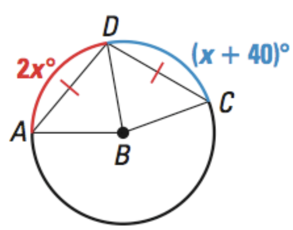
EXAMPLE 4 *Using Theorem 10.4*

You can use Theorem 10.4 to find $m\overset{\frown}{AD}$.

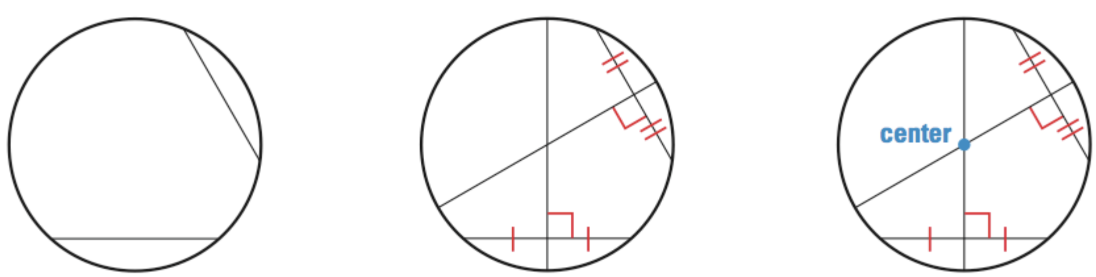
Because $\overline{AD} \cong \overline{DC}$, $\overset{\frown}{AD} \cong \overset{\frown}{DC}$. So, $m\overset{\frown}{AD} = m\overset{\frown}{DC}$.

$2x = x + 40$ **Substitute.**

$x = 40$ **Subtract x from each side.**



Theorem 10.6 can be used to locate a circle's center, as shown below.



1. Draw any two chords that are not parallel to each other.
2. Draw the perpendicular bisector of each chord. These are diameters.
3. The perpendicular bisectors intersect at the circle's center.

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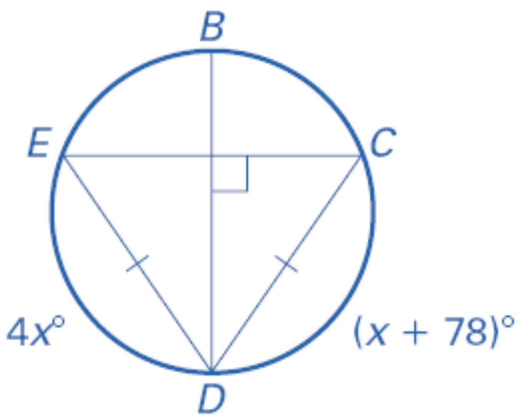
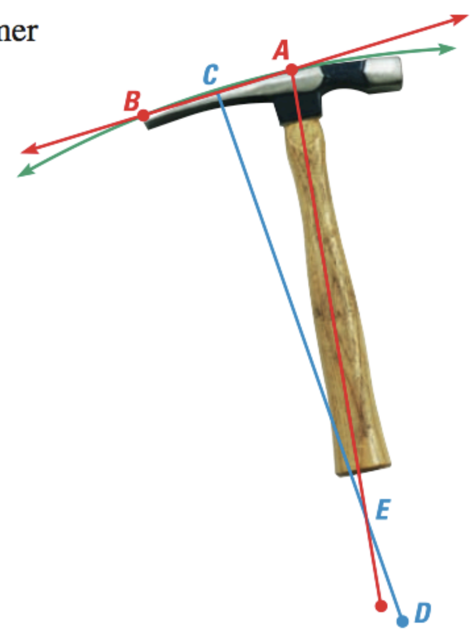
3. Guided Practice: Find $m\overset{\frown}{ED}$.



EXAMPLE 6 *Using Properties of Chords*

MASONRY HAMMER A masonry hammer has a hammer on one end and a curved pick on the other. The pick works best if you swing it along a circular curve that matches the shape of the pick. Find the center of the circular swing.



SOLUTION

Draw a segment $\overline{AB}$, from the top of the masonry hammer to the end of the pick. Find the midpoint C, and draw a perpendicular bisector $\overline{CD}$. Find the intersection of $\overline{CD}$ with the line formed by the handle.

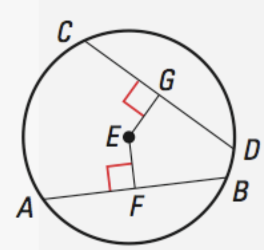
▶ So, the center of the swing lies at E.

THEOREM

THEOREM 10.7

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

$\overline{AB} \cong \overline{CD}$ if and only if $\overline{EF} \cong \overline{EG}$.



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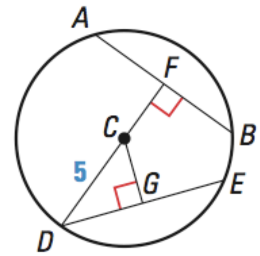
EXAMPLE 7 ***Using Theorem 10.7***

$AB = 8$, $DE = 8$, and $CD = 5$. Find CF.



SOLUTION

Because $\overline{AB}$ and $\overline{DE}$ are congruent chords, they are equidistant from the center. So, $\overline{CF} \cong \overline{CG}$. To find CG, first find DG.

$\overline{CG} \perp \overline{DE}$, so $\overline{CG}$ bisects $\overline{DE}$. Because $DE = 8$, $DG = \frac{8}{2} = 4$.

Then use DG to find CG.

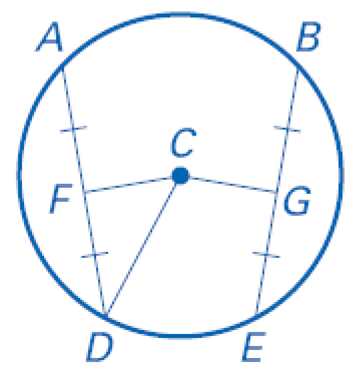
$DG = 4$ and $CD = 5$, so $\triangle CGD$ is a 3-4-5 right triangle. So, $CG = 3$.

Finally, use CG to find CF.

▶ Because $\overline{CF} \cong \overline{CG}$, $CF = CG = 3$.

<u>Guided Practice.</u>

6. If AD = 40 & CD = 25, find CG.



7. Classify each ar as a major arc, a minor arc, or as a semi-circle: $180°$, $67°$, $240°$

8. Find the length of a chord of a circle with radius 8 that is a distance of 5 from the circle.

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9. ______ Find the value of x.

A. 22.5
B. 30
C. 45
D. 60
E. 120